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Finite horizon model predictive control with ellipsoid mapping of uncertain linear systems

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Abstract: A model predictive control scheme was proposed for discrete-time uncertain linear systems subject to input constraints. The cost functional to be minimised is a finite horizon quadratic cost, which describes the performance of the corresponding nominal system. The control action is specified in terms of both feedback and open-loop components. The open-loop part of the control action steers the centre of associated ellipsoids into a set around the origin, while the feedback component forces the actual system states to remain in those ellipsoids. Both feedback and open-loop control are determined online by repeatedly solving a convex optimisation problem. The predictive control scheme guarantees recursive feasibility and robust stability if the convex optimisation problem is feasible at the initial time instant. A numerical example illustrates the effectiveness of the proposed approach.

Nomenclature

The following notations are used throughout the paper. Let $\mathbb{R}$ and $\mathbb{Z}$ denote the field of real number, the set of integer numbers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space.

The notations $\mathbb{Z}_{[c_1,c_2]}$ and $\mathbb{Z}_{[c_1,c_2)}$ are to denote the sets $\{k \in \mathbb{Z} \mid c_1 \leq k \leq c_2\}$ and $\{k \in \mathbb{Z} \mid c_1 \leq k < c_2\}$. For a matrix $M \in \mathbb{R}^{n\times n}$, M^{T} denotes the transpose of M and $\bar{\sigma}(M)$ denotes the largest singular value of matrix M. I_m denotes the $m \times m$ identity matrix, and $*$ denotes the corresponding symmetric block in symmetric matrices.

1 Introduction

Model predictive control (MPC) has received remarkable attention in both practical applications and theoretical research over the last 30 years since it yields optimal performance and it is capable of explicitly dealing with state and input constraints. The basic idea of standard MPC [1–4] is as follows: online, a finite horizon open-loop optimal control problem based on the current measurement of the system states is solved. Then, the first part of the obtained open-loop optimal input trajectory is applied to the system. At the succeeding time instant, the optimal control problem is solved again using new state measurements, and the actual control input is updated.

However, for a nominally stabilising MPC scheme with the presence of disturbances and/or model uncertainties might lead to performance deterioration or even loss of stability [5]. This basically results from two major problems of standard MPC. First, the solution to the optimal control problem is open-loop trajectory, and feedback is only provided at the sampling instants [6]. Second, recursive feasibility often cannot be guaranteed for all admissible uncertainty realisations [7].

An intuitive approach to guarantee robust stability and recursive feasibility is to use a min-max MPC formulation, where the optimal input is determined such that the performance criteria is minimised for a worst-case uncertainty [8–14]. However, such approaches are computationally expensive in general. Furthermore, the optimal input is obtained for a possibly unrealistic worst-case scenario, which often results in poor performance in the case of small actual uncertainties.

For uncertain linear systems, the min–max MPC formulation is circumvented in [15] by repeatedly solving a semi-definite program (SDP) such that an upper bound on the worst-case performance is minimised. This computationally attractive approach is based on the online calculation of robustly positively invariant ellipsoids and associated feedback matrices. The price to pay is a rather small region of attraction.

Many research activities focused on enlarging the region of attraction and/or improving control performance while keeping the computational burden as low as possible.

Using parameter-dependent Lyapunov functions, The authors of [16–18] propose MPC schemes that guarantee asymptotic stability rather than exponential stability, and provide extra degrees of freedom to reduce the conservative of the optimisation problem.

A fixed state-feedback law with perturbations is proposed in [19], where the system trajectory tracks the trajectory related to an a priori fixed state-feedback control law.

A parameter-dependent feedback law in the framework of gain-scheduling is proposed in [20], which offers potential performance improvements compared with approaches with static feedback laws.

By allowing the first control action to be chosen freely, the robust MPC schemes in [21–23] are applicable to systems subject to asymmetric constraints.

Linear parameter-varying (LPV) system is a particular case of linear uncertain systems whose dynamics depend on time-varying parameters. The MPC scheme of LPV systems in [24] is based on ellipsoid mapping over a finite horizon. This approach requires that the rate of the parameter variation is bounded, and is thus restricted to systems with slowly varying parameters. Recently, robust MPC schemes for linear systems with structured feedback uncertainty have been exploited by Smith [25, 26]. The control law is specified in terms of both feedback and open-loop components. The open-loop part steers the trajectory of the nominal system to the origin at the end of the prediction horizon [25] or at some time instants within the prediction horizon [26], whereas the associated feedback law renders some prescribed ellipsoids invariants.

In this paper, we introduce a finite horizon robust MPC scheme of linear systems subject to structured feedback uncertainty and input constraints. The considered finite horizon cost functional, including a terminal function penalising the state at the end of the prediction horizon, solely depends on the trajectories of the nominal system, which is different from the results proposed by the authors of [15, 20, 21], where a worst-case cost functional is minimised. Similar to [25, 26], the idea is to divide the control law in both feedback and open-loop components. The open-loop component steers the nominal trajectories into a nominal terminal set around the origin. The feedback component ensures that the actual state trajectories remain in some associated predicted ellipsoids for any admissible uncertainty realisation. The ellipsoids are calculated such that any perturbed state at the end of the prediction horizon lies in the actual terminal set, which entirely contains the nominal terminal set. The terminal penalty function represents an upper bound on the infinite horizon cost obtained by *fictitiously* applying a linear terminal feedback law, which renders the actual terminal region to be robust positive invariant. The resulting online optimisation problem is formulated as a convex optimisation problem, and the proposed scheme guarantees robust stability and recursive feasibility if the optimisation problem is initially feasible. The terminal set, terminal penalty function, and terminal feedback law are determined by solving linear matrix inequalities (LMIs) offline.

The paper is structured as follows: Section 2 introduces the system considered, and presents some results on ellipsoid mapping, constraints satisfaction and the choice of design parameters. In Section 3, the novel robust MPC scheme is proposed, together with a discussion of its recursive feasibility and asymptotic stability properties. Simulation results are reported in Section 4.

To derive the results proposed in this section, some preliminary results are used. First, we consider the well-known $\mathcal{S}$-procedure.

Lemma 1 ($\mathcal{S}$-procedure for quadratic functions) [27]: Let $F_0, F_1, \ldots, F_p$ be quadratic functions of the variable $\xi \in \mathbb{R}^n$

$$F_i(\xi) := \xi^{\mathrm{T}} T_i \xi + 2\beta_i^{\mathrm{T}} \xi + \delta_i, \quad i \in \mathbb{Z}_{[0,p]}$$

where $\beta_i \in \mathbb{R}^n$, $T_i^{\mathrm{T}} = T_i \in \mathbb{R}^{n\times n}$ and δ_i is a scalar. We consider the following condition on $F_0, \ldots, F_p$

$$F_0(\xi) \geq 0, \quad \textit{for all } \xi \textit{ such that } F_i(\xi) \geq 0, \quad i \in \mathbb{Z}_{[1,p]} \quad (1)$$

If there exist $\tau_i > 0$, for all $i \in \mathbb{Z}_{[1,p]}$, such that for all ξ

$$F_0(\xi) - \sum_{i=1}^{p} \tau_i F_i(\xi) \geq 0$$

then (1) holds. For $p = 1$, the converse holds, provided that there is some ξ_0 such that $F_1(\xi_0) \geq 0$.

We further require the following result.

Lemma 2 [28, 29]: Let $G \in \mathbb{R}^{n\times n}$, $H \in \mathbb{R}^n$, $x \in \mathbb{R}^n$ and $c \in \mathbb{R}$. The inequality

$$x^{\mathrm{T}} G x + 2H^{\mathrm{T}} x + c \leq 0 \quad (2)$$

is satisfied for all $x \in \mathbb{R}^n$, *if and only if*

$$\begin{bmatrix} G & H \\ H^{\mathrm{T}} & c \end{bmatrix} \leq 0 \quad (3)$$

2 Problem setup

Consider the discrete-time linear system with structured feedback uncertainty; see Fig. 1

$$x_{k+1} = Ax_k + Bu_k + B_p p_k \quad (4a)$$

$$q_k = C_q x_k + D_{qu} u_k \quad (4b)$$

$$p_k = \Delta_k q_k \quad (4c)$$

where $x_k \in \mathbb{R}^{n_x}$ is the state, $u_k \in \mathcal{U} \subseteq \mathbb{R}^{n_u}$ is the control input, and $p_k \in \mathbb{R}^{n_p}$ and $q_k \in \mathbb{R}^{n_p}$ describe the structured feedback uncertainty of the considered system. The input constraint set is defined as

$$\mathcal{U} := \{u \in \mathbb{R}^{n_u} \mid g_j^{\mathrm{T}} u \leq h_j, \quad j \in \mathbb{Z}_{[1,r_u]}\} \quad (5)$$

where $r_u \in \mathbb{Z}_{[0,\infty)}$ is the number of input constraints, $g_j \in \mathbb{R}^{n_u \times 1}$ and $h_j \in \mathbb{R}$. Denote

$$\Xi := \Big\{ \Delta \in \mathbb{R}^{n_p \times n_p} \mid \Delta := \mathrm{diag}\,[\Delta_1, \Delta_2, \ldots \Delta_m], \; \bar{\sigma}(\Delta_l) \leq 1, \Delta_l \in \mathbb{R}^{n_l \times n_l}, l \in \mathbb{Z}_{[1,m]}, \sum_{l=1}^{m} n_l = n_p \Big\} \quad (6)$$

where Δ_l is a repeated scalar or a full block. Δ_l models a number of factors, such as non-linearities, dynamics or

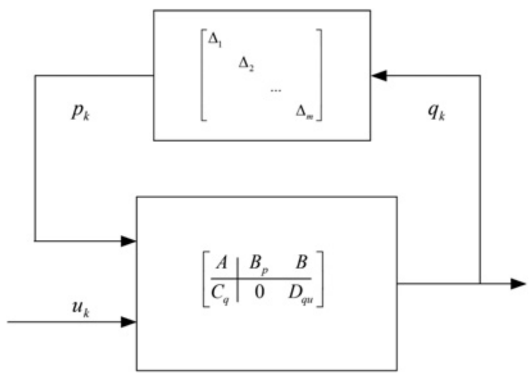

Fig. 1 *Graphical representation of structured feedback uncertainty*

parameters, that are unknown, unmodelled or neglected. The operator $\Delta_k \in \Xi$ is a block diagonal matrix.

Denote the projection onto the lth component associated with $\triangle_l$ as Π_l, that is, Π_l satisfies $\triangle_l = \Pi_l \Delta, \Delta \in \Xi$. Then, the norm bound on each $\triangle_l$ implies that

$$(\Pi_l p_k)^{\mathrm{T}} \Pi_l p_k \leq (\Pi_l q_k)^{\mathrm{T}} \Pi_l q_k, \quad l \in \mathbb{Z}_{[1,m]} \tag{7}$$

Remark 1: The uncertainty formulation can also be viewed as replacing the state-space matrices (A, B) by $(A, B) \in (\mathcal{A}, \mathcal{B})$, where $(\mathcal{A}, \mathcal{B}) := \{A + B_p \Delta C_q, B + B_p \Delta D_{qu} \mid \Delta \in \Xi\}$.

The nominal dynamics of system (4) are defined by

$$z_{k+1} := Az_k + Bv_k \tag{8}$$

where $z_k \in \mathbb{R}^{n_x}$ is the nominal state and $v_k \in \mathbb{R}^{n_u}$ is the nominal control input.

Assumption 1: The system state x_k can be measured in real-time and the pair (A, B) is stabilisable.

The goal of this paper is to design an MPC control law, which steers the system trajectory from the state x_k at time k to the equilibrium such that constraints (5) are satisfied. For this, the control signal is specified as

$$u_k = K_k(x_k - z_k) + v_k \tag{9}$$

where $K_k \in \mathbb{R}^{n_u \times n_x}$, for all $k \in \mathbb{Z}_{[1,\infty)}$.

Define the error $e_k := x_k - z_k$ as the deviation of the actual state from the nominal state. Under control (9), we obtain the error dynamics

$$e_{k+1} = (A + BK_k)e_k + B_p p_k \tag{10a}$$

$$q_k = (C_q + D_{qu}K_k)e_k + C_q z_k + D_{qu} v_k \tag{10b}$$

$$p_k = \triangle_k q_k \tag{10c}$$

Ellipsoids centred around the nominal trajectory z_k are defined as

$$\mathbb{P}(z_k) := \left\{ x \in \mathbb{R}^{n_x} \mid (x - z_k)^{\mathrm{T}} P(x - z_k) \leq \frac{\alpha}{4} \right\} \tag{11}$$

where positive-definite matrix $P \in R^{n_x \times n_x}$, and scalar $\alpha > 0$ are given.

By a modification of a result in [26], we first introduce a preliminary lemma, called the ellipsoid mapping, which can cast the actual system trajectory inside the ellipsoids centred along the nominal trajectory.

Lemma 3 (Ellipsoid mapping): Consider system (4) and the block diagonal perturbation constraints (7). Let $z_k \in \mathbb{R}^{n_x}$, $x_k \in \mathbb{P}(z_k)$, $P \in \mathbb{R}^{n_x \times n_x}$ is a positive-definite matrix and $\alpha > 0$. Suppose that there exist $v_k \in \mathbb{R}^{n_u}$, $K_k \in \mathbb{R}^{n_u \times n_x}$, $\xi_k \in [0, 1)$, $\Lambda_k = \mathrm{diag}(\lambda_{1k} I, \ldots, \lambda_{mk} I)$ with $\lambda_{ik} > 0$ for all $i \in \mathbb{Z}_{[1,m]}$, such that

$$S_k := \begin{bmatrix} -\xi_k P & * & * & * & * \\ 0 & -\Lambda_k & * & * & * \\ 0 & 0 & \dfrac{\alpha}{4}(\xi_k - 1) & * & * \\ A + BK_k & B_p \Lambda_k & 0 & -P^{-1} & * \\ C_q + D_{qu} K_k & 0 & C_q z_k + D_{qu} v_k & 0 & -\Lambda_k \end{bmatrix} \leq 0 \tag{12}$$

Then, the control law $u_k = K_k(x_k - z_k) + v_k$ guarantees that $x_{k+1} \in \mathbb{P}(z_{k+1})$, where $z_{k+1} = Az_k + Bv_k$.

Proof: Under the control $u_k = K_k(x_k - z_k) + v_k$, the requirement that $x_{k+1} \in \mathbb{P}(z_{k+1})$ is equivalent to the quadratic functional

$$T_0 = \begin{bmatrix} x_k - z_k \\ p_k \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} (A + BK_k)^{\mathrm{T}} \\ B_p^{\mathrm{T}} \end{bmatrix} P [A + BK_k \quad B_p] \times \begin{bmatrix} x_k - z_k \\ p_k \end{bmatrix} - \frac{\alpha}{4} \leq 0 \tag{13}$$

The requirement that $x_k \in \mathbb{P}(z_k)$ is equivalent to

$$T_1 = \begin{bmatrix} x_k - z_k \\ p_k \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} I \\ 0 \end{bmatrix} P [I \quad 0] \begin{bmatrix} x_k - z_k \\ p_k \end{bmatrix} - \frac{\alpha}{4} \leq 0 \tag{14}$$

Each of the m perturbation constraints (7) is equivalent to

$$\begin{aligned} T_{2l} = {} & \begin{bmatrix} x_k - z_k \\ p_k \end{bmatrix}^{\mathrm{T}} \left(\begin{bmatrix} 0 \\ I \end{bmatrix} \Pi_l^{\mathrm{T}} \Pi_l [0 \quad I] - \begin{bmatrix} (C_q + D_{qu} K_k)^{\mathrm{T}} \\ 0 \end{bmatrix} \right. \\ & \left. \Pi_l^{\mathrm{T}} \Pi_l [(C_q + D_{qu} K_k) \quad 0] \right) \begin{bmatrix} x_k - z_k \\ p_k \end{bmatrix} - 2(C_q z_k + D_{qu} v_k)^{\mathrm{T}} \\ & \Pi_l^{\mathrm{T}} \Pi_l [(C_q + D_{qu} K_k) \quad 0] \begin{bmatrix} x_k - z_k \\ p_k \end{bmatrix} \\ & - (C_q z_k + D_{qu} v_k)^{\mathrm{T}} \Pi_l^{\mathrm{T}} \Pi_l (C_q z_k + D_{qu} v_k) \leq 0 \end{aligned} \tag{15}$$

Via the $\mathcal{S}$-Procedure, the requirement of the lemma is met if there exists ξ_k and Λ_k such that

$$T_0 - \xi_k T_1 - \sum_{l=1}^{m} \lambda_{lk}^{-1} T_{2l} \leq 0 \tag{16}$$

The quadratic function (16) is a functional of $[x_k - z_k \quad p_k]^{\mathrm{T}}$, which is required to hold for all x_k and p_k. With the Schur complement and Lemma 2, and some simple matrix transformations, (16) is reduced to the matrix constraints $S_k \leq 0$. □

Remark 2: Pre- and post-multiplying (12) by $[I \quad 0 \quad 0 \quad I \quad 0]$ and $[I \quad 0 \quad 0 \quad I \quad 0]^{\mathrm{T}}$, we obtain

$$\begin{bmatrix} -\xi_k P & * \\ A + BK_k & -P^{-1} \end{bmatrix} \leq 0$$

Using the Schur complement, this is equivalent to

$$(A + BK_k)^{\mathrm{T}} P (A + BK_k) - \xi_k P \leq 0$$

Since $\xi_k \in [0, 1)$, this implies that the nominal system (8) is asymptotically stable under the control law $v_k := K_k z_k$.

Note that (12) represents an LMI since S_k is linear in its unknowns. Lemma 3 proposes a way to calculate a feedback law K_k and an open-loop input v_k such that $x_{k+1} \in \mathbb{P}(z_{k+1})$ if $x_k \in \mathbb{P}(z_k)$. The associated control input satisfies constraints (5) if the conditions stated in the following lemma are satisfied.

Lemma 4 (Input constraints) [26]: The control signal u_k satisfies the input constraints (5) for all $x_k \in \mathbb{P}(z_k)$, if and only if there exist $v_k \in \mathbb{R}^{n_u}$, $K_k \in \mathbb{R}^{n_u \times n_x}$, and $\eta_{k,j} > 0$, for

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all $j \in \mathbb{Z}_{[1,r_u]}$ such that

$$U_{k,j} := \begin{bmatrix} -\eta_{k,j}P & * \\ g_j^{\mathrm{T}}K_k & \frac{\alpha}{4}\eta_{k,j} + 2g_j^{\mathrm{T}}v_k - 2h_j \end{bmatrix} \leq 0, \quad \forall j \in \mathbb{Z}_{[1,r_u]} \tag{17}$$

Remark 3: Note that Lemma 4 does not introduce any conservativeness because of its condition being '*if and only if*'.

The following lemma states conditions for the calculation of P and α, that were assumed to be given in the results above.

Lemma 5 (Parameters P and α) [15]: Consider system (4) and the block diagonal perturbation constraints (7). Let $Q \in \mathbb{R}^{n_x \times n_x}$ and $R \in \mathbb{R}^{n_u \times n_u}$ be positive-definite matrices. If there exist $\Lambda_a = \mathrm{diag}(\lambda_{1a}I, \ldots, \lambda_{ma}I)$ with $\lambda_{ia} > 0$, for all $i \in Z_{[1,m]}$, positive-definite matrix $X \in \mathbb{R}^{n_x \times n_x}$, non-square matrix $Y \in \mathbb{R}^{n_u \times n_x}$, and $\alpha > 0$, such that

$$\begin{bmatrix} -X & * & * & * & * \\ R^{\frac{1}{2}}Y & -\alpha & * & * & * \\ Q^{\frac{1}{2}}X & 0 & -\alpha & * & * \\ C_qX + D_{qu}Y & 0 & 0 & -\Lambda_a & * \\ AX + BY & 0 & 0 & 0 & -X + B_p\Lambda_aB_p^{\mathrm{T}} \end{bmatrix} \leq 0 \tag{18a}$$

$$\begin{bmatrix} h_jh_j^{\mathrm{T}} & g_j^{\mathrm{T}}Y \\ * & X \end{bmatrix} \geq 0 \tag{18b}$$

with $j \in Z_{[1,r_u]}$, then, the ellipsoid

$$\mathcal{X}_f := \{x \in \mathbb{R}^{n_x} \mid x^{\mathrm{T}}Px \leq \alpha\} \tag{19}$$

with $P := \alpha X^{-1}$, and the linear state feedback control law $u_k = Fx_k$ with $F := YX^{-1}$ have the following properties:

(1) Let $M(x_k) := x_k^{\mathrm{T}}Px_k$. Then, $M(x_{k+1}) - M(x_k) \leq -x_k^{\mathrm{T}}Qx_k - u_k^{\mathrm{T}}Ru_k$ for all $k \in \mathbb{Z}_{[0,\infty)}$ and for all $x_k \in \mathcal{X}_f$;
(2) $\mathcal{X}_f$ is robustly invariant for system (4) controlled by the feedback control law $u_k = Fx_k$;
(3) $u_k = Fx_k \in \mathcal{U}$ for all $x_k \in \mathcal{X}_f$.

Define an ellipsoid Ω_z as

$$\Omega_z := \left\{z \in \mathbb{R}^{n_x} \mid z^{\mathrm{T}}Pz \leq \frac{\alpha}{4}\right\}$$

which will serve as the terminal set of the online optimisation problem in the following section.

If we choose $v_k = Fz_k$, and if the additional constraint stated in the next lemma is satisfied, the solution of (18a) also satisfies (12), and therefore, Lemma 3 is satisfied for all $z_k \in \Omega_z$. This result plays an important role in the construction of a feasible solution to the proposed MPC scheme later.

Lemma 6: Let $\mathcal{W} := (C_q + D_{qu}F)^{\mathrm{T}}\alpha\Lambda_a^{-1}(C_q + D_{qu}F)$, and let X, Y, α and Λ_a be a feasible solution to (18a). Suppose there exists $\xi \in [0, 1)$ such that

$$\begin{bmatrix} (1-\xi)P - \mathcal{W} & \mathcal{W} \\ \mathcal{W} & F^{\mathrm{T}}RF + Q - (1-\xi)P \end{bmatrix} \geq 0 \tag{20}$$

where $P = \alpha X^{-1}$ and $F = YX^{-1}$. Then, X, Y, α, ξ and Λ_a also satisfy (12) for all $z_k \in \Omega_z$ with $\Lambda_k := \alpha^{-1}\Lambda_a$, $K_k := F$, $\xi_k := \xi$ and $v_k := Fz_k$.

Proof: By the Schur complement, (18a) is equivalent to

$$\begin{bmatrix} F^{\mathrm{T}}RF + Q - P + \mathcal{W} & * & * \\ 0 & -\alpha\Lambda_a^{-1} & * \\ A + BF & B_P & -P^{-1} \end{bmatrix} \leq 0 \tag{21}$$

and (12) is equivalent to

$$\begin{bmatrix} -\xi P + \mathcal{W} & * & * & * \\ 0 & -\alpha\Lambda_a^{-1} & * & * \\ z_k^{\mathrm{T}}\mathcal{W} & 0 & \frac{\alpha}{4}(\xi - 1) + z_k^{\mathrm{T}}\mathcal{W}z_k & * \\ A + BF & B_P & 0 & -P^{-1} \end{bmatrix} \leq 0 \tag{22}$$

with $v_k = Fz_k$.

Obviously, $\mathcal{W}^{\mathrm{T}} = \mathcal{W} > 0$. Define $\beta := \frac{\alpha}{4}(\xi - 1) + z_k^{\mathrm{T}}\mathcal{W}z_k$, and assume that $\beta < 0$, then (22) can be rewritten as

$$\begin{bmatrix} -\xi P + \mathcal{W} - \beta^{-1}\mathcal{W}z_kz_k^{\mathrm{T}}\mathcal{W} & * & * \\ 0 & -\alpha\Lambda_a^{-1} & * \\ A + BF & B_P & -P^{-1} \end{bmatrix} \leq 0 \tag{23}$$

Suppose that

$$F^{\mathrm{T}}RF + Q - P + \mathcal{W} \geq -\xi P + \mathcal{W} - \beta^{-1}\mathcal{W}z_kz_k^{\mathrm{T}}\mathcal{W}, \quad \forall z_k \in \Omega_z \tag{24}$$

Then, satisfaction of (21) implies that (23) holds, that is, the solution of (14a) also satisfies (12). Since $\beta < 0$, using the Schur complement, (24) is equivalent to

$$\begin{bmatrix} F^{\mathrm{T}}RF + Q - (1-\xi)P & * \\ z_k^{\mathrm{T}}\mathcal{W} & \frac{\alpha}{4}(1-\xi) - z_k^{\mathrm{T}}\mathcal{W}z_k \end{bmatrix} \geq 0 \tag{25}$$

which means that

$$z_k^{\mathrm{T}}(\mathcal{W} + \mathcal{W}(F^{\mathrm{T}}RF + Q - (1-\xi)P)^{-1}\mathcal{W})z_k \leq \frac{\alpha}{4}(1-\xi), \quad \forall z_k \in \Omega_k \tag{26}$$

Note that $\beta < 0$ is satisfied automatically if inequality (25) has a feasible solution.

Since $z_k \in \Omega_k$, that is, $z_k^{\mathrm{T}}Pz_k \leq \frac{\alpha}{4}$, it follows that if we impose

$$\mathcal{W} + \mathcal{W}(F^{\mathrm{T}}RF + Q + (\xi - 1)P)^{-1}\mathcal{W} \leq P(1-\xi) \tag{27}$$

which is the Schur complement to (20), then (26), and thus, (24), holds. Therefore (21) implies (23), which completes the proof. □

Unfortunately, (20) is a non-LMI, which is not easy to solve. Thus, normally we solve the LMIs (18a and b), and then check whether or not the parameters Λ_a, P, F satisfy (20).

Based on the foregoing lemmas, in the next section we propose a novel finite horizon MPC scheme and discuss its feasibility and stability properties if (20) is satisfied.

3 Finite horizon MPC with ellipsoid mapping

In this section, we present the main result of this paper, namely a robust MPC scheme for the uncertain linear system (4). The control sequence obtained consists of two components: a nominal open-loop control sequence and a feedback control law. The idea is that the nominal control sequence steers the centre of predicted ellipsoids into a prescribed nominal terminal set, where the centres of the ellipsoids are generated by the nominal model (8). The feedback control law ensures that the state of the uncertain linear system lies in the ellipsoids for all admissible uncertainties. The open-loop component is similar to that of [24, 25, 30], but in our scheme the feedback control law is defined by an online determined time-varying matrix rather than a static one. As it will be shown, both the nominal control action and the feedback control law are obtained *online* by repeatedly solving a convex optimisation problem. Suppose that P and α are obtained offline by Lemma 5. To distinguish the actual state and predicted trajectories, in what follows the index $k+i/k$ denotes future values at time $k+i$ predicted at time k, $i \in \mathbb{Z}_{[0,N-1]}$.

Denote the sequences $\mathbf{z}_k := \{z_{k/k}, z_{k+1/k}, \ldots, z_{k+N-1/k}\}$ and $\mathbf{v}_k := \{v_{k/k}, v_{k+1/k}, \ldots, v_{k+N-1/k}\}$, and define the nominal cost functional as

$$J(\mathbf{z}_k, \mathbf{v}_k) = \sum_{i=0}^{N-1}\{z_{k+i/k}^{\mathrm{T}} Q z_{k+i/k} + v_{k+i/k}^{\mathrm{T}} R v_{k+i/k}\} + z_{k+N/k}^{\mathrm{T}} P z_{k+N/k} \tag{28}$$

where $N \in \mathbb{Z}_{[0,\infty)}$ is the prediction horizon, $Q \in \mathbb{R}^{n_x \times n_x}$ and $R \in \mathbb{R}^{n_u \times n_u}$ are positive-definite weighting matrices, $z_{k+i/k}^{\mathrm{T}} Q z_{k+i/k} + v_{k+i/k}^{\mathrm{T}} R v_{k+i/k}$ is the stage cost and $z_{k+N/k}^{\mathrm{T}} P z_{k+N/k}$ is the terminal penalty function.

The proposed MPC scheme is based on the repeated solution of the following optimisation problem for the current state $x_k \in \mathbb{R}^{n_x}$:

Problem 1:

$$\underset{z_k, v_{k+i/k}, K_{k+i/k}, \xi_{k+i/k}, \eta_{k+i,j/k} \Lambda_{k+i/k}}{\text{minimise}} \quad J(\mathbf{z}_k, \mathbf{v}_k) \tag{29a}$$

subject to

$$z_{k+i+1/k} = A z_{k+i/k} + B v_{k+i/k}, \quad z_{k/k} = z_k \tag{29b}$$

$$x_k \in \mathbb{P}(z_k) \tag{29c}$$

$$S_{k+i/k} \leq 0, \quad i \in \mathbb{Z}_{[0,N-1]} \tag{29d}$$

$$U_{k+i/k,j} \leq 0, \quad i \in \mathbb{Z}_{[0,N-1]}, \quad j \in \mathbb{Z}_{[1,r_u]} \tag{29e}$$

$$z_{k+N/k} \in \Omega_z \tag{29f}$$

where $\Lambda_{k+i/k} = \mathrm{diag}(\lambda_{1,k+i/k} I, \ldots, \lambda_{m,k+i/k} I)$, with $\lambda_{l,k+i/k} > 0$, $l \in \mathbb{Z}_{[1,m]}$, $\eta_{k+i,j/k} > 0$ and $\xi_{k+i/k} \in [0, 1)$, $i \in \mathbb{Z}_{[0,N-1]}$. The notation $S_{k+i/k}$ and $U_{k+i/k,j}$ denote the term S_k in Lemma 3 and $U_{k,j}$ in Lemma 4, respectively, at the future time $k+i$ predicted at time k.

Remark 4: Since P and α are determined *a priori*, Problem 1 is a convex optimisation problem [31]. Furthermore, the optimisation problem considered can be transformed into an SDP since (29c and f) can be rewritten as LMIs by the Schur complement. Thus, for simplicity, in what follows we refer to Problem 1 as an SDP.

Remark 5: In this paper, only input constraint (5) is considered. In order to take state constraint into account, similar to [32], a restricted constraint on the ellipse centres has to be estimated *offline* and involved in Problem 1.

We refer to the set Ω_z as the 'nominal' terminal region of Problem 1. The following lemma states that any feasible solution to Problem 1 steers the *actual* system state into the set $\mathcal{X}_f$ in N steps. Since $\mathcal{X}_f$ has been obtained using Lemma 5, we know that it is robustly invariant for the original system (4). Obviously, $\mathcal{X}_f$ represents the 'actual' terminal region of the proposed scheme for system (4) with the static control law $u_k = F x_k$, which motivates us to use this control law as a candidate terminal control law, while $x_k \in \mathcal{X}_f$.

Lemma 7: Let $\mathbf{z}_k$ and $\mathbf{v}_k$ denote a feasible solution to Problem 1. Then, the admissible state trajectory $\mathbf{x}_k := \{x_{k/k}, x_{k+1/k}, \ldots, x_{k+N/k}\}$ satisfies $x_{k+N/k}^{\mathrm{T}} P x_{k+N/k} \leq \alpha$, that is, $x_{k+N/k}^{\mathrm{T}} \in \mathcal{X}_f$.

Proof: Since $x_k \in \mathbb{P}(z_k)$, it follows by repeatedly exploiting Lemma 3 that $x_{k+N/k} \in \mathbb{P}(z_{k+N/k})$, that is

$$(x_{k+N/k} - z_{k+N/k})^{\mathrm{T}} P (x_{k+N/k} - z_{k+N/k}) \leq \frac{\alpha}{4}$$

Note that for any $\beta_1, \beta_2 \in \mathbb{R}^{n_x}$ and positive-definite matrix $M \in \mathbb{R}^{n_x \times n_x}$, $\beta_1^{\mathrm{T}} M \beta_1 + \beta_2^{\mathrm{T}} M \beta_2 \geq \frac{1}{2}(\beta_1 + \beta_2)^{\mathrm{T}} M (\beta_1 + \beta_2)$, which can be confirmed by a simple inner-product transformation. In virtue of this, we conclude that $z_{k+N/k}^{\mathrm{T}} P z_{k+N/k} + (x_{k+N/k} - z_{k+N/k})^{\mathrm{T}} P (x_{k+N/k} - z_{k+N/k}) \geq \frac{1}{2} x_{k+N/k}^{\mathrm{T}} P x_{k+N/k}$.

Since $z_{k+N/k}^{\mathrm{T}} P z_{k+N/k} \leq \frac{\alpha}{4}$ and $(x_{k+N/k} - z_{k+N/k})^{\mathrm{T}} P (x_{k+N/k} - z_{k+N/k}) \leq \frac{\alpha}{4}$, then $x_{k+N/k}^{\mathrm{T}} P x_{k+N/k} \leq \alpha$. □

Remark 6: Let $b > 0$ and $c > 0$ such that $\frac{1}{b} + \frac{1}{c} = \frac{1}{2}$. Define the sets $\mathbb{P}_0(z_k) := \{x \mid (x_k - z_k)^{\mathrm{T}} P (x_k - z_k) \leq \frac{\alpha}{b}\}$ and $\Omega_0 := \{z_k \mid z_k^{\mathrm{T}} P z_k \leq \frac{\alpha}{c}\}$. If Problem 1 has a feasible solution, then, by similar arguments as in the proof of Lemma 7 we can guarantee that $x_{k+N/k}^{\mathrm{T}} P x_{k+N/k} \leq \alpha$ also holds. In other words, $\mathbb{P}_0(z_k)$ and Ω_0 can serve as the mapping ellipsoid and the 'nominal' terminal set, respectively, of Problem 1.

We are now ready to present the robust MPC scheme that is based on the ellipsoid mapping, and establish its feasibility and robust stability properties.

First, we assume that the following assumption is satisfied.

Assumption 2: Let the pair (P, α) be a feasible solution to (18), and suppose that this pair also satisfies (20).

The robust MPC control law is derived from the solution of the convex optimisation Problem 1, which is solved repeatedly at each sampling instant k based on the state measurement x_k.

Algorithm 1: Step 1. At time instant k, measure the state x_k and solve Problem 1.
Step 2. Apply $u_k = v_k + K_{k/k}(x_k - z_k)$ to the actual system (4). Set $k = k+1$ and go to Step 1.

The feasibility and stability properties of the proposed scheme according to Algorithm 1 depend on the feasibility of Problem 1 at the initial time instant.

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Let z_k^* and $v_{k+i/k}^*, K_{k+i/k}^*, \xi_{k+i/k}^*, \Lambda_{k+i/k}^*, \eta_{k+i,j/k}^*$, $i \in \mathbb{Z}_{[0,N-1]}$ be the optimal solution to Problem 1. For brevity of notation, denote $\Gamma_{k+i/k}^* := [v_{k+i/k}^*, K_{k+i/k}^*, \xi_{k+i/k}^*, \Lambda_{k+i/k}^*, \eta_{k+i,j/k}^*]$ for all $i \in \mathbb{Z}_{[0,N-1]}$.

Theorem 1 (Feasibility): Problem 1 is feasible at all time instants if it is feasible at $k = 0$.

Proof: Assume that Problem 1 is feasible at time instant k, and its solution is

$$[z_k^*, \Gamma_{k/k}^*, \Gamma_{k+1/k}^*, \ldots, \Gamma_{k+N-1/k}^*] \tag{30}$$

Denote the actual state sequence and the nominal state sequence, related to (30), as $[x_{k+1/k}, x_{k+2/k}, \ldots, x_{k+N/k}]$ and $[z_{k+1/k}^*, z_{k+2/k}^*, \ldots, z_{k+N/k}^*]$. According to Algorithm 1, the input applied to the system (4) at time k is $u_k = K_{k/k}^*(x_k - z_k^*) + v_{k/k}^*$. By Lemma 3, the input guarantees that the actual system state x_{k+1} lies in the ellipsoid $\mathbb{P}(z_{k+1/k}^*)$ for any admissible uncertainty.

Denote $\Gamma_{k+N/k+1} := [Fz_{k+N/k}^*, F, \xi, \Lambda_a, \eta]$. With state x_{k+1} at time instant $k+1$, consider the following feasible solution candidate

$$[z_{k+1/k}^*, \Gamma_{k+1/k}^*, \ldots, \Gamma_{k+N-1/k}^*, \Gamma_{k+N/k+1}] \tag{31}$$

where F and Λ satisfy (18), and ξ and η will be introduced later.

Based on Lemma 7 and $z_{k+N/k}^* \in \Omega_z$, we know that $x_{k+N/k} \in \mathcal{X}_f$ and $Fx_{k+N/k}$ satisfies the input constraints (5), that is, $g_j^{\mathrm{T}} Fx_{k+N/k} \le h_j$. Obviously, $Fx_{k+N/k} = Fz_{k+N/k}^* + F(x_{k+N/k} - z_{k+N/k}^*)$. From the proof of Lemma 4, it follows that if there exists $\eta > 0$, such that

$$\begin{aligned} &2g_j^{\mathrm{T}} F(x_{k+N/k} - z_{k+N/k}^*) + 2g_j^{\mathrm{T}} v_{k+N/k} - 2h_j \\ &\quad - \eta\left((x_{k+N/k} - z_{k+N/k}^*)^{\mathrm{T}} P(x_{k+N/k} - z_{k+N/k}^*) - \frac{\alpha}{4}\right) \le 0 \end{aligned}$$

then, inequality (17) is satisfied. In terms of $v_{k+N/k+1} := Fz_{k+N/k}^*$ while $z_{k+N/k}^* \in \Omega_z$, this is equivalent to

$$\begin{aligned} &2g_j^{\mathrm{T}} Fx_{k+N/k} - 2h_j \\ &\quad - \eta\left((x_{k+N/k} - z_{k+N/k}^*)^{\mathrm{T}} P(x_{k+N/k} - z_{k+N/k}^*) - \frac{\alpha}{4}\right) \le 0 \end{aligned} \tag{32}$$

Since $g_j^{\mathrm{T}} Fx_{k+N/k} \le h_j$, and $(x_{k+N/k} - z_{k+N/k}^*)^{\mathrm{T}} P(x_{k+N/k} - z_{k+N/k}^*) \le \frac{\alpha}{4}$, which results from Lemma 5, obviously, there exists a scalar $\eta > 0$ satisfying condition (32). Therefore at time instant $k+1$, constraint (29e) is satisfied with $v_{k+N/k+1}$, $\eta_{k+N/k+1} := \eta$ and $K_{k+N/k+1} := F$.

Based on Assumption 2 and Lemma 6, there exists ξ such that constraint (29d) is satisfied with $\xi_{k+N/k+1} := \xi$, $v_{k+N/k+1}$, $K_{k+N/k+1}$ and $\Lambda_{k+N/k+1} := \alpha^{-1}\Lambda_a$.

F is a feasible control law, which robustly stabilises system (4), for all $x \in \mathcal{X}_f$. Further, it is an asymptotically stable control law for the nominal system (8), too. Therefore $z_{k+N+1/k+1} := A + BFz_{k+N/k}^*$ satisfies (29f).

Based on the above discussion, sequence (31) is a feasible solution to Problem 1 at time instant $k+1$. □

Let us define a Lyapunov function candidate as

$$V(x_k) := \min_{z_k, v_{k+i/k}, K_{k+i/k}, \xi_{k+i/k}, \eta_{k+i,j/k}\Lambda_{k+i/k}} J(\mathbf{z}_k, \mathbf{v}_k) \tag{33}$$

Here, we emphasise that the optimal value of the cost functional is solely defined by the state x_k, which is measured online.

Theorem 2 (Stability): Suppose that Problem 1 is feasible at time $k = 0$. Then, system (4) is asymptotically stabilised under the proposed MPC control law according to Algorithm 1.

Proof: (1) $0 < V(x_k) < +\infty$ for all $x_k \neq 0$, which follows directly from the definition of $V(\cdot)$.
(2) $V(0) = 0$, which is confirmed by choosing all the terms of sequences $\mathbf{z}_k$ and $\mathbf{v}_k$ as 0.
(3) Assume that Problem 1 is feasible at time instant k, and its solution is given by (30). Then

$$V(x_k) = \sum_{i=0}^{N-1} z_{k+i/k}^{*T} Q z_{k+i/k}^* + v_{k+i/k}^{*T} R v_{k+i/k}^* + z_{k+N/k}^{*T} P z_{k+N/k}^* \tag{34}$$

Denote $\mathbf{z}_{k+1} := \{z_{k+1/k+1}, z_{k+2/k+1}, \ldots, z_{k+N/k+1}\}$ and $\mathbf{v}_{k+1} := \{v_{k+1/k+1}, v_{k+2/k+1}, \ldots, v_{k+N/k+1}\}$. Since (31) is a feasible solution to Problem 1 at time instant $k+1$, we have

$$\begin{aligned} J(\mathbf{z}_{k+1}, \mathbf{v}_{k+1}) &= \sum_{i=0}^{N-1} z_{k+i+1/k+1}^{\mathrm{T}} Q z_{k+i+1/k+1} + v_{k+i+1/k+1}^{\mathrm{T}} \\ &\quad \times R v_{k+i+1/k+1} + z_{k+N+1/k+1}^{\mathrm{T}} P z_{k+N+1/k+1} \\ &= \sum_{i=1}^{N-1} z_{k+i/k}^{*T} Q z_{k+i/k}^* + v_{k+i/k}^{*T} R v_{k+i/k}^* \\ &\quad + z_{k+N/k}^{*T} (Q + F^{\mathrm{T}} R F) z_{k+N/k}^{\mathrm{T}} \\ &\quad + z_{k+N+1/k+1}^{\mathrm{T}} P z_{k+N+1/k+1} \end{aligned} \tag{35}$$

Owing to the Principle of Optimality, we have $V(x_{k+1}) \le J(\mathbf{z}_{k+1}, \mathbf{v}_{k+1})$. Thus

$$\begin{aligned} V(x_{k+1}) - V(x_k) &\le J(\mathbf{z}_{k+1}, \mathbf{v}_{k+1}) - V(x_k) \\ &= z_{k+N/k}^{*T}(Q + F^{\mathrm{T}} R F) z_{k+N/k}^* + z_{k+N+1/k+1}^{\mathrm{T}} \\ &\quad \times P z_{k+N+1/k+1} - z_{k/k}^{*T} Q z_{k/k}^* - v_{k/k}^{*T} R v_{k/k}^* \\ &\quad - z_{k+N/k}^{\mathrm{T}} P z_{k+N/k} \end{aligned} \tag{36}$$

Since $z_{k+N+1/k+1}^{\mathrm{T}} P z_{k+N+1/k+1} - z_{k+N/k}^{\mathrm{T}} P z_{k+N/k} \le -z_{k+N/k}^{*T}(Q + F^{\mathrm{T}} R F) z_{k+N/k}^*$, which results from Lemma 5, $V(x_{k+1}) - V(x_k) \le -z_{k/k}^{*T} Q z_{k/k}^* - v_{k/k}^{*T} R v_{k/k}^*$.

Clearly, $V(x_k)$ is a Lyapunov function and thus, system (4) is asymptotically stabilised [2] by the control (9). □

Remark 7: Problem 1 is based on the prediction of the future nominal trajectory associated with the nominal system (8). Although an exact prediction of the actual trajectory is not possible in the presence of uncertainties, we know that the actual system trajectory lies in the prescribed ellipsoids centred around the nominal trajectory with respect to any admissible uncertainty.

Assumption 2 is strong and plays an important role in the construction of feasible solution to Problem 1, as well as in the proof of stability of the proposed finite horizon MPC scheme.

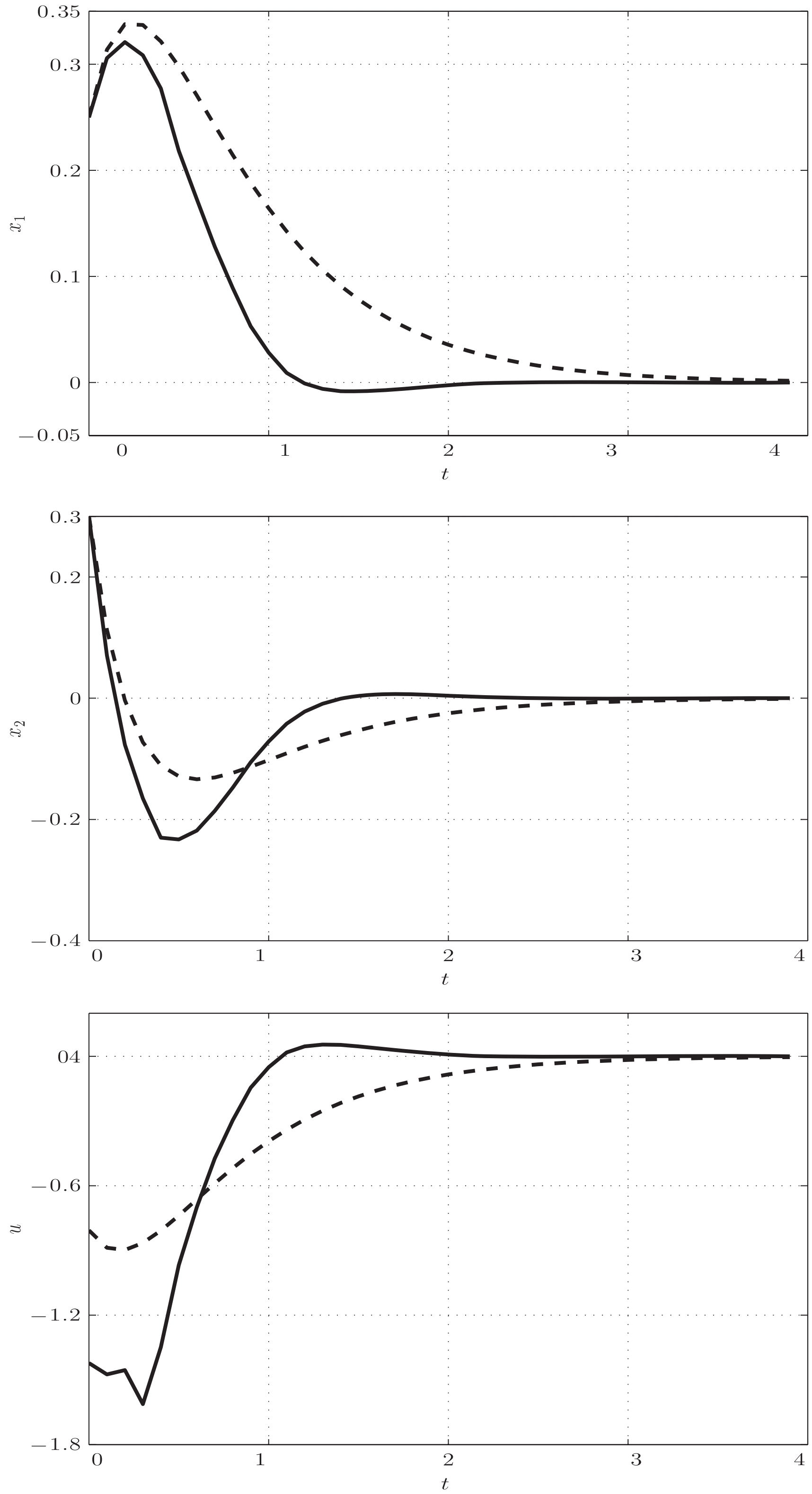

Fig. 2 *Comparing of dynamic responses and input trajectory for the initial state $x_0 = [0.25\ 0.30]^{\mathrm{T}}$, solid line: proposed finite horizon MPC with ellipsoid mapping according to Algorithm 1, dashed line: closed-loop MPC [15]*

4 Numerical example

Consider the discrete-time linear system

$$x_{1,k+1} = x_{1,k} + (0.15 + 0.125\rho_k)x_{2,k}$$
$$x_{2,k+1} = 0.15x_{1,k} + (0.5 - 0.125\rho_k)x_{2,k} + 0.1\kappa u_k$$

subject to the input constraint $[0.5 \quad -0.5]^{\mathrm{T}} u_k \le [1 \quad 1]^{\mathrm{T}}$, where $\kappa = 0.8$ and $x_{j,k}$, $j = 1, 2$, is the jth element of vector x_k. The time-varying parameter ρ_k is bounded by $\rho_k \in [-1 \quad 1]$, for all $k \in \mathbb{Z}_{[0,\infty)}$. Thus, in the notations of (4), $A = \begin{bmatrix} 1 & 0.15 \\ 0.15 & 0.5 \end{bmatrix}$, $B = [0 \quad 0.1\kappa]^{\mathrm{T}}$, $B_p = [0.25 \quad -0.25]^{\mathrm{T}}$, $C_q = [0 \quad 0.5]$ and $D_{qu} = 0$. The uncertainty is described by

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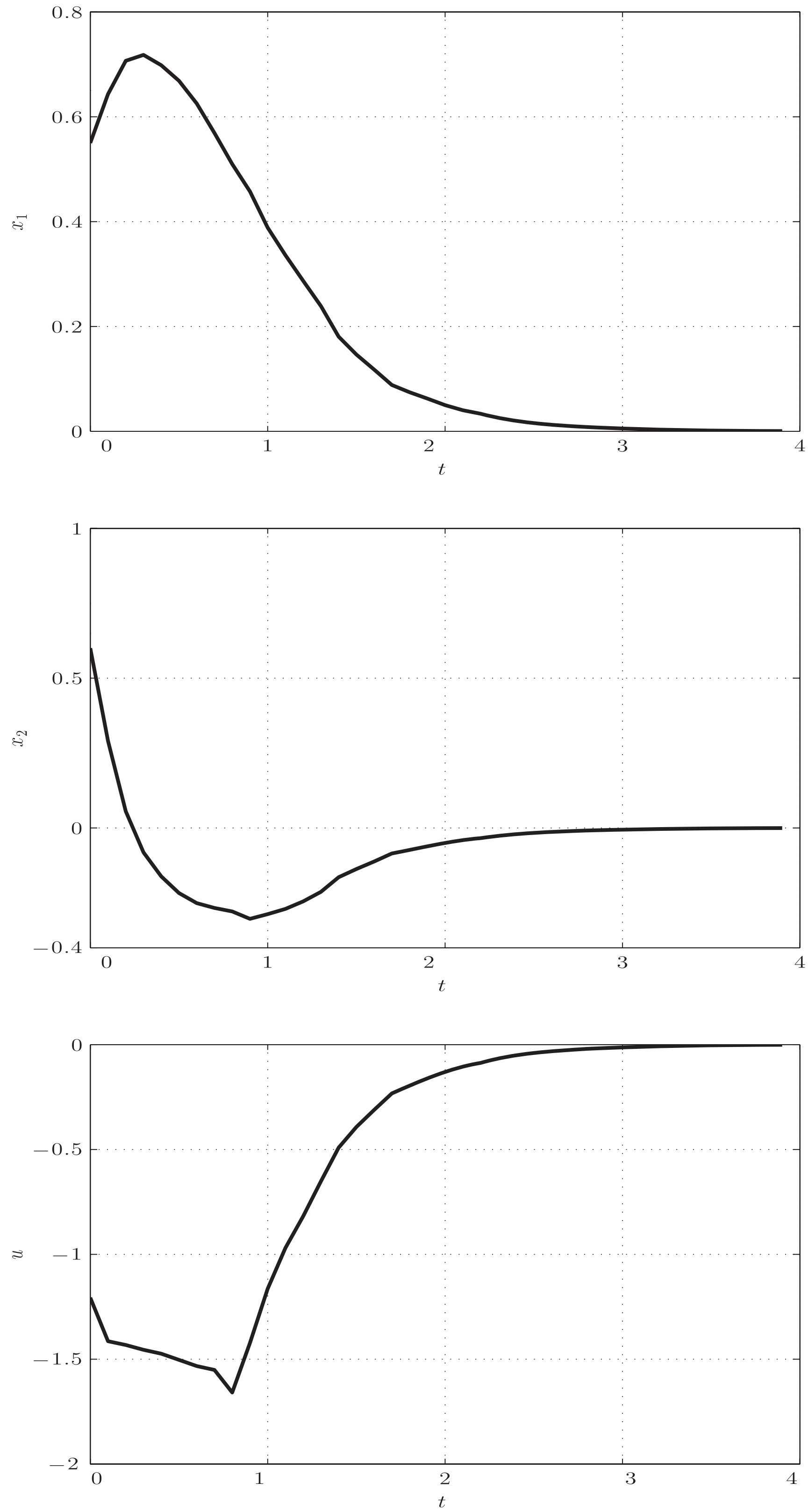

Fig. 3 *Exemplary time profiles for dynamic responses and input trajectory with Algorithm 1 from the initial state* $x_0 = [0.55\ 0.60]^T$

$\Xi = \{A + B_p\rho_k C_q : \rho_k \in [-1 \quad 1]\}$. We choose the matrices $Q = \mathrm{diag}(1, 1)$ and $R = 1$ in the cost functional (28).

The matrix $P = \left[\begin{smallmatrix} 1059.7 & 219.8 \\ 219.8 & 171.3 \end{smallmatrix}\right]$ and the scalar $\alpha = 50$ are obtained by solving (18). Furthermore, P and α do satisfy Assumption 2. Thus, we use the finite horizon MPC with ellipsoid mapping according to Algorithm 1.

Exemplary, Fig. 2 shows simulation results for the initial state $x_0 = [0.25 \quad 0.30]^{\mathrm{T}}$ corresponding to $\rho_k = 0.5$, for all $k \in \mathbb{Z}_{[0,\infty)}$. The solid line shows the state and input trajectories obtained by the proposed method with prediction horizon $N = 10$. The dashed line shows the trajectories obtained by [15]. The performance of the proposed finite horizon MPC with ellipsoid mapping is worse than the one of [15] since only *nominal* performance rather than *robust* performance [15] is minimised in Problem 1.

However, for $x_0 = [0.55 \quad 0.60]^{\mathrm{T}}$ and $x_0 = [0.65 \quad 0.75]^{\mathrm{T}}$, the closed-loop MPC scheme [15] has no feasible solution at the initial time instant, whereas the proposed finite horizon MPC scheme guarantees recursive feasibility and stability with the prediction horizons $N = 38$ and $N = 48$, respectively. This shows that the proposed finite horizon MPC scheme has a different region of attraction compared with [15]. Since part of the admissible input is used to keep the actual state in the ellipsoids around the nominal trajectory, a large prediction horizon is required in the proposed finite horizon MPC with ellipsoid mapping. Fig. 3 shows the simulation results for the initial state $x_0 = \left[0.55 \quad 0.60\right]^{\mathrm{T}}$, whereas $\rho_k \equiv 0.5$. The simulation shows that stability as well as constraint satisfaction are guaranteed even if the initial state is far from the equilibrium.

5 Conclusions

In this paper, we proposed an MPC scheme for discrete-time linear systems with structured feedback uncertainty and constraints. The control signal is constructed by both feedback and open-loop terms, which are calculated online by solving a convex optimisation problem. The open-loop component steers the centre of associated ellipsoids into a terminal set, while the feedback component keeps the system state in those ellipsoids for all admissible uncertainties. If the optimisation problem is initially feasible, the proposed MPC strategy guarantees recursive feasibility and closed-loop stability. A simulation example illustrated the effectiveness of the derived theory.

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7 References

1 Chen, H., Allgöwer, F.: 'A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability', *Automatica*, 1998, **34**, (10), pp. 1205–1217
2 Mayne, D.Q., Rawlings, J.B., Rao, C.V., Scokaert, P.O.M.: 'Constrained model predictive control: stability and optimality', *Automatica*, 2000, **36**, (6), pp. 789–814
3 Magni, L., Nicolao, G.D., Scattolini, R.: 'A stabilizing model-based predictive control algorithm for nonlinear systems', *Automatica*, 2001, **37**, (9), pp. 1351–1362
4 Fontes, F.A.C.C.: 'A general framework to design stabilizing nonlinear model predictive controllers', *Syst. Control Lett.*, 2001, **42**, (2), pp. 127–143
5 Grimm, G., Messina, M.J., Tuna, S., Teel, A.R.: 'Examples when nonlinear model predictive control is nonrobust', *Automatica*, 2004, **40**, (10), pp. 1729–1738
6 Findeisen, R.: 'Nonlinear model predictive control: a sampled-data feedback perspective'. PhD thesis, University of Stuttgart, Germany, 2004
7 Chisci, L., Rossiter, J.A., Zappa, G.: 'Systems with persistent disturbances: predictive control with restricted constraints', *Automatica*, 2001, **37**, (7), pp. 1019–1028
8 Scokaert, P.O.M., Mayne, D.Q.: 'Min-max feedback model predictive control for constrained linear systems', *IEEE Trans. Autom. Control*, 1998, **43**, (8), pp. 1136–1142
9 Bemporad, A., Borrelli, F., Morari, M.: 'Min-max control of constrained uncertain discrete-time linear systems', *IEEE Trans. Autom. Control*, 2003, **48**, (9), pp. 1600–1606
10 Chen, H., Scherer, C.W., Allgöwer, F.: 'A game theoretic approach to nonlinear robust receding horizon control of constrained systems'. Proc. American Control Conf., Albuquerque, New Mexico, 1997, pp. 3073–3077
11 Magni, L., De Nicolao, G., Scattolini, R., Allgöwer, F.: 'Robust model predictive control for nonlinear discrete-time systems', *Int. J. Robust Nonlinear Control*, 2003, **13**, (3–4), pp. 229–246
12 Fontes, F.A.C.C., Magni, L.: 'Min-max model predictive control of nonlinear systems using discontinuous feedbacks', *IEEE Trans. Autom. Control*, 2003, **48**, (10), pp. 1750–1755
13 Limon, D., Alamo, T., Salas, F., Camacho, E.F.: 'Input-to-state stability of min–max MPC controllers for nonlinear systems with bounded uncertainties', *Automatica*, 2006, **42**, (5), pp. 797–803
14 Raimondo, D.M., Limon, D., Lazar, M., Magni, L., Camacho, E.F.: 'Min–max model predictive control of nonlinear systems: a unifying overview on stability', *Eur. J. Control*, 2009, **15**, (1), pp. 5–21
15 Kothare, M.V., Balakrishnan, V., Morari, M.: 'Robust constrained model predictive control using linear matrix inequalities', *Automatica*, 1996, **32**, (10), pp. 1361–1379
16 Lee, S., Won, S.: 'Model predictive control for linear parameter varying systems using a new parameter dependent terminal weighting matrix', *IEICE Trans. Fundamentals*, 2006, **E89-A**, (8), pp. 2166–2172
17 Wada, N., Saito, K., Saeki, M.: 'Model predictive control for linear parameter varying systems using parameter dependent Lyapunov function', *IEEE Trans. Autom. Control*, 2006, **53**, (12), pp. 1446–1450
18 Cuzzola, F., Geromel, J.C., Morari, M.: 'An improved approach for constrained robust model predictive control', *Automatica*, 2002, **38**, (7), pp. 1183–1189
19 Kouvaritakis, B., Rossiter, J.A., Schuurmans, J.: 'Efficient robust predictive control', *IEEE Trans. Autom. Control*, 2000, **45**, (8), pp. 1545–1549
20 Yu, S.-Y., Böhm, C., Chen, H., Allgöwer, F.: 'Stabilizing model predictive control for LPV systems subject to constraints with parameter-dependent control law'. Proc. American Control Conf., St. Louis, Missouri, 2009, pp. 3118–3123
21 Lu, Y., Arkun, Y.: 'Quasi-min–max MPC algorithms for LPV systems', *Automatica*, 2000, **36**, (4), pp. 527–540
22 Park, B.G., Lee, J.W., Kwon, W.H.: 'Robust one-step receding horizon control for constrained systems', *Int. J. Robust Nonlinear Control*, 1999, **9**, (7), pp. 381–395
23 Yu, S.-Y., Böhm, C., Chen, H., Allgöwer, F.: 'MPC with one free control action for constrained LPV systems'. Proc. IEEE Int. Conf. Control Applications, Yokohama, Japan, 2010, pp. 1343–1348
24 Suzuki, H., Sugie, T.: 'Model predictive control for linear parameter varying constrained systems using ellipsoidal set prediction', *Int. J. Control*, 2007, **80**, (2), pp. 314–321
25 Smith, R.S.: 'Robust model predictive control of constrained linear systems'. Proc. American Control Conf., Boston, MA, 2004, pp. 245–250
26 Smith, R.S.: 'Model predictive control of uncertain constrained linear systems: LMI based methods'. Technical Report CUED/F-INFENG/TR.462, Department of Engineering, University of Cambridge, UK, 2006
27 Boyd, S., El Ghaoui, L., Feron, E., Balakishnan, V.: 'Linear matrix inequalities in system and control theory' (SIAM, Philadelphia, 1994)
28 Hilbert, D.: 'Über die Darstellung definiter Formen als Summe von Formenquadraten', *Math. Ann.*, 1888, **32**, pp. 342–350.
29 Dullerud, G., Smith, R.: 'A nonlinear functional approach to LFT model validation', *Syst. Control Lett.*, 2002, **47**, (1), pp. 1–11
30 Yu, S.-Y., Böhm, C., Chen, H., Allgöwer, F.: 'Robust model predictive control with disturbance invariant sets'. Proc. American Control Conf., Baltimore, MD, 2010, pp. 6262–6267
31 Boyd, S., Vandenberghe, L.: 'Convex optimization' (Cambridge University Press, Cambridge, UK, 2004)
32 Marruedo, D.L., Alamo, T., Camacho, E.F.: 'Input-to-state stable MPC for constrained discrete-time nonlinear systems with bounded additive uncertainties'. Proc. 41th IEEE Conf. Decision Control, 2002, pp. 4619–4624

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